mass of the electron cannot arise entirely from electromagnetic interactions, which conserve both parity and chirality. But it is useless for the neutrino, and we are forced to conclude that only perturbation theory can account for its zero mass.

XI. CONCLUSIONS

The Feynman rules for massless particles in the $(2j+1)$-component formalism are identical with those derived in Ref. 1 for particles of mass $m>0$. It is only necessary to pass to the limit $m\to 0$ to obtain the correct propagators for internal lines, and wave functions for external lines. Also, the various possible invariant Hamiltonians $\mathcal{H}(x)$ can be constructed out of the fields $\varphi_\sigma(x)$ and $\chi_\sigma(x)$, with no distinction between massive and massless particle fields.

Furthermore, the transformation properties of $\varphi_\sigma(x)$ and $\chi_\sigma(x)$ under **T**, **C**, and **P** are the same for $m>0$ and $m=0$. If **P** and/or **C** are conserved it is very convenient to unite $\varphi_\sigma(x)$ and $\chi_\sigma(x)$ into a $2(2j+1)$-component field $\psi(x)$, which transforms according to the reducible $(j,0)\oplus(0,j)$ representation; for $j=\frac{1}{2}$ this yields the Dirac formalism, while for $j=1$ it corresponds to the union of the irreducible fields $\mathbf{E}\pm i\mathbf{B}$ into a six-vector $\{\mathbf{E},\mathbf{B}\}$. Here again there is no distinction to be made between zero and nonzero mass, so we need not repeat here the details of the $2(2j+1)$-component formalism[10] constructed in Ref. 1.

We have seen no hint of anything like gauge invariance in our work so far. In fact, the really significant distinctions between field theories for zero and nonzero mass arise when we try to go beyond the $(2j+1)$- or $2(2j+1)$-component formalisms. In particular, for $m>0$ there is no difficulty in constructing tensor fields transforming according to the $(j/2,j/2)$ representations, while for $m=0$ this is strictly forbidden by the theorem proven in Sec. III. We will see in a forthcoming article that the attempt to evade this prohibition and yet keep the S matrix Lorentz-invariant yields all the results usually associated with gauge invariance.

PHYSICAL REVIEW VOLUME 134, NUMBER 4B 25 MAY 1964

Possible Effects of Strong Interactions in Feinberg-Pais Theory of Weak Interactions. II

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(Received 23 December 1963)

In a previous paper, a simplified model was used to study the effects of strong interactions on the weak interaction theory of Feinberg and Pais. In this paper, we use a more general argument, a power count based upon the Ward-Takahashi-Nishijima multimeson vertex function identity, to show that the same conclusion remains valid even when crossed ladder graphs are included. Our conclusion may not apply, however, to the modified program of peratization where $W-W$ scattering plays an essential role.

I. INTRODUCTION

IN a previous paper,[1] the possible effects of strong interactions on the peratization theory of Feinberg and Pais[2] were studied in a simplified model where the strong interactions acted through modifications only of the baryon vertices and propagators. It was shown there that the final "peratized" nuclear vector β-decay coupling strength $G_\beta{}^v$ is no longer equal to the "peratized" μ-decay coupling strength, G_μ if the vector current is conserved. In this paper, we wish to present an argument which shows that the same power counting conclusion holds when all possible effects of strong interactions, within the framework of peratization theory, are taken into account. Furthermore, the very nature of our argument shows that the same conclusion holds even when one includes, in peratization theory, the sum over the crossed ladder graphs so long as power counting is valid. That is to say, if we define the peratized (crossed+uncrossed ladder graphs) μ-decay constant by $G_\mu=(g^2/m^2)(1-\eta)$, then the corresponding peratized nuclear vector β-decay constant is $G_\beta{}^v=(g^2/m^2)(1-Z\eta)$, where Z is the strong interaction nucleon renormalization factor. Thus, unless peratization vanishes ($\eta=0$) when all graphs are included, the situation remains that $G_\mu\neq G_\beta{}^v$ when the vector current is conserved. This makes it hard to understand the

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* Supported in part by a grant from the National Science Foundation.
† Boese predoctoral fellow.

[1] N. P. Chang, Phys. Rev. **133**, B454 (1964).
[2] G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963); **133**, B477 (1964).

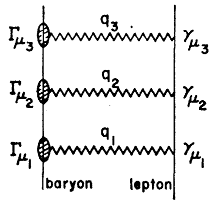

FIG. 1. A typical uncrossed ladder graph.

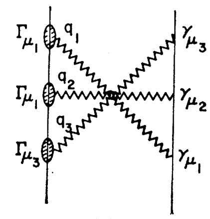

FIG. 2. A typical crossed ladder graph.

approximate ($\sim 2\%$) equality between G_μ and $G_\beta{}^\nu$ that has been observed.

Since our argument is essentially a power count, the conclusion does not apply to the recent modifications to the peratization program, as reported by Professor Pais at the 1963 Sienna Conference,[3] where W-W scattering is taken into account in an essential way leading to an inverse expansion in ln$g$.

In Sec. II, we introduce the concept of symmetrization necessary for the generalized Ward-Takahashi-Nishijima[4] identity, and make direct use of the identity to prove our major assertion. In Sec. III, we give a corresponding identity involving only proper graphs. This new identity leads, of course, ultimately to the same conclusion already reached in Sec. II.

II. PERATIZATION AND W-T IDENTITY

Peratization is the result of a summation (suitably defined) of the leading (unrenormalizable) divergences of each order of weak interaction graph. Thus far, the peratization theory as formulated by Feinberg and Pais, has included only the so-called uncrossed ladder graphs (as illustrated in Fig. 1). The hope is to include soon some of the crossed ladder graphs as well (as illustrated in Fig. 2). In any case, the key feature of the peratization program lies in retaining in the summation only the $q_\mu q_\nu$ part of the vector-meson propagator $\Delta_{\mu\nu}(q)=(\delta_{\mu\nu}+q_\mu q_\nu/m^2)/(q^2+m^2)$. This feature is also the key to the whole argument to be presented here. For it means that one needs only to know about the asymptotic behavior of $q_\mu q_\nu' \cdots q_\rho'' \Gamma_{\mu\nu\cdots\rho}(p;q,q',\cdots q'')$ rather than $\Gamma_{\mu\nu\cdots\rho}(p;q,q',\cdots,q'')$ itself, where $\Gamma_{\mu\nu\cdots\rho}$ is the multimeson baryon-baryon vertex function (Fig. 3). And for this, a generalized Ward-Takahashi (W-T) identity is eminently useful and entirely adequate. We begin, then, with a study of the general W-T type identity.

Let $W_\mu(x)$ be the renormalized field operator for the charged vector meson, $(W_\mu{}^* \equiv (W_i{}^\dagger, -W_4{}^\dagger))$, satisfying

$$(\Box_x - m^2)W_\mu(x) = gJ_\mu{}^-(x)\,, \tag{1}$$

where $J_\mu{}^-$ is the weak vector current to which W_μ is coupled in the Lagrangian. $\partial_\mu J_\mu{}^- = 0$ by assumption. (We suppress the axial vector current for the moment.)

To lowest order in g, but to all orders on strong interactions, we need consider only the T product of the current operators. This is clear from the general expression for S-matrix element, when one separates out the part involving the baryons, the strongly interacting particles.

We have, thus, to deal only with the derivates of T products involving the weak currents. In particular, we have to study the object (see Appendix)

$$\frac{\partial}{\partial y_{1\mu}}\frac{\partial}{\partial y_{2\nu}}\cdots\frac{\partial}{\partial y_{m\lambda}}\frac{\partial}{\partial z_{1\rho}}\frac{\partial}{\partial z_{2\sigma}}\frac{\partial}{\partial z_{m+1\tau}}\,,$$
$$T(\psi_n(x)\bar{\psi}_\rho(x')J_\mu{}^\dagger(y_1)J_\nu{}^\dagger(y_2)\cdots J_\lambda{}^+(y_m)\,,$$
$$J_\rho{}^-(z_1)J_\sigma{}^-(z_2)\cdots J_\tau(z_{m+1}))\,.$$

In momentum space this object reads correspondingly

$$\begin{aligned}&(q_1)_\mu(q_2)_\nu\cdots(q_m)_\lambda(q_1')_\rho(q_2')_\nu\cdots(q'_{m+1})\,,\\ &S_F(p-\textstyle\sum q-\sum q')\Gamma_{\mu\nu\cdots\lambda\rho\sigma\cdots}(p;q_1,q_2,\cdots q_1',q_2'\cdots)\,,\\ &S_F(p)\,,\end{aligned} \tag{2}$$

where S_F is the nucleon propagator.

Let us distinguish between the vector meson polarization indices and the four-momentum indices. Clearly, the object

$$(q_1)_\mu(q_2)_\nu\cdots(q_1')_\rho(q_2')_\sigma\cdots$$

is completely symmetric under any permutation of the set of indices

$$(q_1,\mu),\ (q_2,\nu),\ \cdots,\ (q_1',{}_\rho),\ (q_2',\sigma),\ \cdots.$$

Thus, if one symmetrically sums over q_1, q_2, $\cdots$, one would need to consider only the completely symmetric part of $\Gamma_{\mu\nu}\cdots(p;q_1,q_2,\cdots)$ in the generalized W-T identity. For the counting of powers, it is sufficient to consider the symmetric limit, i.e., we set $q_1=q_2=\cdots=q_1'=q_2'=\cdots=\Lambda$ and let $\Lambda\to\infty$.

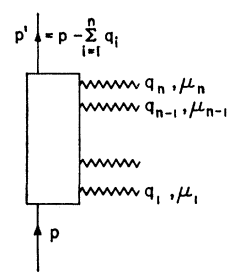

FIG. 3. The full n-meson baryon-baryon vertex function, $\Gamma_{\mu_1\cdots\mu_n}(p;q_1\cdots q_n)$.

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[3] A. Pais, Proceedings of the Sienna Conference, 1963 (unpublished).

[4] K. Nishijima, Phys. Rev. **119**, 485 (1960).

In coordinate space, the correctly symmetrized T product to use is

$$(1/\sigma_{2m+1})\{T(\psi_n(x)\bar{\psi}_p(x')F_\mu(y_1)F_\nu(y_2)\cdots \times F_\lambda(y_m)F_\rho(z_1)F_\sigma(z_2)\cdots F(z_{m+1}))\}\,, \quad (3)$$

where

$$F_\mu(y)\equiv J_\mu{}^+(y)+J_\mu{}^-(y)$$

and

$$\sigma_{2m+1}=\binom{2m+1}{m}=(2m+1)!/m!(m+1)!.$$

The many additional terms in the above T product do not contribute because of charge conservation. The ones that do are precisely the permuted versions of the original T product. Note that this symmetrization is with respect to mesons line of opposite charges. The T product, as written, of course, includes automatically all permutations of meson lines of the same charge family. There is thus an over-all symmetry irrespective of charge lines. In other words, the current $F_\mu(y)$, after symmetrization, "looks" like a neutral current.

The generalized W-T identity for this T-product vertex function can be derived following Nishijima. Thus we note that, analogous to the photon case,

$$[F_4(x),F_\nu(y)]_{x_0=y_0}=0\,,$$
$$[F_4(x),\psi_n(y)]_{x_0=y_0}=-\psi_p(y)\delta^3(x-y)\,. \quad (4)$$

It follows that the final identity reads

$$\partial_\mu\partial_\nu\cdots\partial_\lambda\partial_\rho\partial_\sigma\cdots\partial_\tau\frac{1}{\sigma_{2m+1}}\langle T(\psi_n(x)\bar{\psi}_p(x')F_\mu(y_1)F(y_2)\cdots F_\lambda(y_m)F_\rho(z_1)F_\sigma(z_2)\cdots F_\tau(z_{m+1}))\rangle_0$$
$$=\frac{(-i)^{2m+1}}{\sigma_{2m+1}}\delta(y_1-x)\delta(y_2-x)\cdots\delta(z_{m+1}-x)\langle T(\psi_p(x)\bar{\psi}_p(x'))\rangle_0$$
$$-\sum P(y_1y_2\cdots z_{m+1})\delta(y_1-x')\delta(y_2-x)\cdots\delta(z_{m+1}-x)\langle T(\psi_n(x)\bar{\psi}_n(x^1))\rangle_0$$
$$+\sum P(y_1y_2\cdots z_{m+1})\delta(y_1-x')\delta(y_2-x')\delta(y_3-x)\cdots\delta(z_{m+1}-x)\langle Y(\psi_p(x)\bar{\psi}_p(x^1))\rangle_0+\cdots$$
$$-\delta(y_1-x')\delta(y_2-x')\delta(y_3-x')\cdots\delta(z_{m+1}-x')\langle T(\psi_n(x)\bar{\psi}_n(x'))\rangle_0\,, \quad (5)$$

where $\sum P(\cdots)$ stands for a sum over all distinct permutations of the vectors $y_1y_2\cdots z_{m+1}$ in the string of δ functions.

The detailed structure of this identity is irrelevant for this section. In momentum space we translate it to read for the symmetric limit $q_1=q_2=\cdots=\Lambda$ and find

$$S_F(p-(2m+1)\Lambda)\Lambda_\mu\Lambda_\nu\cdots\Lambda_\tau\,{}_{\mu\nu\cdots\tau}(p;\Lambda,\Lambda,\cdots\Lambda)S_F(p) =(1/\sigma_{2m+1})S_F(p)+O(1/\Lambda)\,,$$

or, equivalently, as $\Lambda\rightarrow\infty$

$$\Lambda_\mu\Lambda_\nu\cdots\Lambda_\tau\Gamma_{\mu\nu\cdots\tau}(p;\Lambda,\Lambda,\cdots\Lambda) =(1/\sigma_{2m+1})\{S_F{}^{-1}(p-(2m+1)\Lambda)+O(1)\}\,. \quad (6)$$

The asymptotic behavior of this function can now be written as

$$\Lambda_\mu\Lambda_\nu\cdots\Lambda_\tau\,{}_{\mu\nu\cdots\tau}(;\Lambda,\Lambda,\cdots,\Lambda) =-Z(i\gamma,\Lambda)[m!(m+1)!/(2m)!]\,. \quad (7)$$

This last statement, one recalls, follows from Lehmann's theorem on the spectral representation of the nucleon propagator function.

The meaning of this nth meson Ward-Takahashi identity that we have derived becomes clear when we examine a simple nth meson-ladder rung vertex graph, where all the intermediate states are poles. We evaluate it to lowest order in g and in strong interactions. We find that its contraction with $q_1,q_2,\cdots$ gives

$$g^n\gamma\cdot q_nS_F{}^0\left(k-\sum_1^{n-2}q_i\right)\gamma\cdot q_{n-1} \times S_F{}^0\left(k-\sum_1^{n-1}q_i\right)\cdots\gamma\cdot q_2S_F{}^0(k-q_1)\gamma\cdot q_1\,,$$

which in the symmetric asymptotic limit reads

$$g^n\gamma\cdot q[1/(n-1)!]$$

or, for $n=2m+1$

$$g^{2m+1}\gamma\cdot q[1/(2m)!]\,.$$

Upon adding the additional graphs which are permutations, within the same charge family, of the vector meson lines we reproduce very easily the result (e.g., m positive, $m+1$ negative mesons)

$$g^{2m+1}\gamma\cdot q[m!(m+1)!/(2m)!]\,.$$

Now we can understand the true significance of a generalized W-T identity. It says that the asymptotic behavior of a full nth meson-vertex function (Fig. 3) along the momenta axes is entirely determined by the lowest order graphs.

The modifications to the internal lines due to strong interactions simply disappear asymptotically in this symmetric limit. The Z factor in the identity comes from the renormalization of the initial and final baryon lines.

As a result of this identity, the leading divergences, as far as peratization is concerned, in each nth-order graph remain the same, except that for a Z factor coming from the external line renormalization. This is true for uncrossed as well as crossed ladder graphs and is true even for more general graphs. We have assumed, as in the previous paper, that the axial vector current is also conserved in the high-energy limit.

This is the basis for writing down the statement given in the Introduction.

III. W-T IDENTITIES FOR PROPER GRAPHS

An alternate, though less direct, way of arriving at the conclusion of Sec. II involves a study of the generalized W-T indentities for proper graphs. Because of their intrinsic interest as identities, we reproduce here the arguments leading to them in some detail.

By proper vertex graphs we mean, as is conventional, those graphs which cannot be disconnected by cutting any single internal-nucleon line. Furthermore, in what follows, we deal only with those vertex graphs which are symmetrized as already indicated in (3). For clarity of presentation, we write down, for illustration, the symmetrized improper two-meson vertex function (Fig. 4)

$$\Gamma_{\mu_2}(p-q_1;q_2)S_F(p-q_1)\Gamma_{\mu_1}(p;q_1)+\Gamma_{\mu_1}(p-q_2;q_1)S_F(p-q_2)\Gamma_{\mu_2}(p;q_2). \quad (8)$$

If we call $V_{\mu_1\cdots\mu_n}(p;q_1\cdots q_n)$ the proper, symmetrized, vertex function for n-meson-baryon vertex graph, then

Theorem 1 ($n>1$)

$$(q_1)_{\mu_1}V_{\mu_1\cdots\mu_n}(p;q_1\cdots q_n)=V_{\mu_2\cdots\mu_n}(p-q_1;q_2\cdots q_n)-V_{\mu_2\cdots\mu_n}(p;q_2\cdots q_n). \quad (9)$$

Proof: We prove this by induction. We observe first that the corresponding statement for the full vertex function reads

$$(q_1)_{\mu_1}S_F\left(p-\sum_{i=1}^{n}q_i\right)\Gamma_{\mu_1\cdots\mu_n}(p;q_1\cdots q_n)S_F(p)$$
$$=S_F\left(p-\sum_{1}^{n}q_i\right)\Gamma_{\mu_2\cdots\mu_n}(p-q_1;q_2\cdots q_n)S_F(p-q_1)$$
$$-S_F\left(p-\sum_{i=2}^{n}q_i\right)\Gamma_{\mu_2\cdots\mu_n}(p;q_2\cdots q_n)S_F(p). \quad (10)$$

This follows immediately from Eq. (3).

Now it can be directly verified by using (10) and (8) that the identity (9) is true for $n=2$. By actual enumeration, (Fig. 5) which is entirely straightforward, for the general n case, assuming it true for $n-1$ the theorem follows.

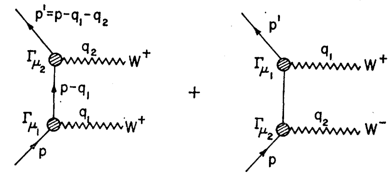

FIG. 4. An example of the "meson" symmetrization referred to in the text.

Theorem 2. ($n>1$)

$$(q_1)_{\mu_1}\cdots(q_n)_{\mu_n}V_{\mu_1\cdots\mu_n}(p;q_1\cdots q_n)$$
$$=+\Bigg[S_F^{-1}(p)-\sum_{i=1}^{n}S_F^{-1}(p-q_1)+\sum_{i<j}S_F^{-1}(p-q_i-q_j)+\cdots+(-)^n\sum_{i_1<i_2<\cdots<i_n}S_F^{-1}(p-q_{i_1}-q_{i_2}-\cdots-q_{i_n})\Bigg].$$

This theorem follows from the previous theorem by induction.

An interesting consequence of this theorem is its asymptotic limit. Using the asymptotic behavior of $S_F^{-1}(q)\rightarrow Z(i\gamma\cdot q)$, we find the leading behavior as all the q_i's go to infinity to be given by

$$Z\left\{\sum_{i=1}^{n}\mathbf{q}_i-\sum_{i_1<i_2}^{n}(\mathbf{q}_{i_1}+\mathbf{q}_{i_2})+\cdots+(-)^n\sum_{i_1<i_2<\cdots<i_n}^{n}(\mathbf{q}_{i_1}+\cdots+\mathbf{q}_{i_n})\right\}$$

($\mathbf{q}_i\equiv i\gamma\cdot q_i$) which vanishes at the symmetric limit ($q_1=q_2=\cdots=\Lambda\rightarrow\infty$). This means that the asymptotic behavior of the proper graphs is one power less than the improper "pole" graphs. The "pole" graphs are precisely the graphs which involve exclusively nucleon propagators and single-meson vertices already considered in paper I. This also confirms the result derived in Sec. II.

IV. CONCLUSIONS

While the conclusions presented here may not be completely rigorous, we believe they indicate the general nature of the final results. Thus, it seems to us the main point of this discussion is that the generalized Ward-Takahashi identity rules out any modifications

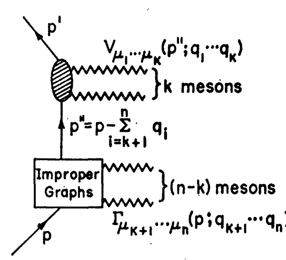

FIG. 5. A typical improper n-meson vertex graph.

in the internal lines of the leading divergence graphs. The external Z-renormalization factor is the only change. For the lowest order graph, the simple unrenormalized baryon-baryon-meson vector vertex function has a Z^{-1} factor which comes from the pole of the unrenormalized S_F propagator. That is why the lowest order graph does not have an extra Z. This difference of Z factors between the first-order graph and higher order leading divergent graphs underlies the break of equality of peratization.

This break of equality leads to the situation where a conserved current implies an unequal $G_\beta{}^\nu$ and G_μ. Such a situation is not easily amenable to reconciliation with experiment if one insists on vector current conservation. Perhaps when the full import of peratization is understood, the approximate equality between $G_\beta{}^\nu$ and G_μ will be restored, with a conserved vector current. Recent progress by Pais in the improvement of peratization program may yet restore such equality.

ACKNOWLEDGMENTS

One of us (N. P. C.) wishes to thank Professor R. Oppenheimer for his kind hospitality. The other (H. S. M.) wishes to thank Professor G. Feinberg for encouragement in this problem.

APPENDIX

In this Appendix we discuss further the fine points about writing down the general W-meson vertex function. In particular, we show the connection between the T product in coordinate space and the Feynman graph for vertex function in momentum space.

We begin with the T product

$$\langle T(\psi_n(x)\bar{\psi}_p(x')W_\mu{}^*(y_1)W_\nu{}^*(y_2)\cdots W_\lambda{}^*(y_m)W_\rho(z_1)W_\sigma(z_2)\cdots W_\tau(z_{m+1}))\rangle_0.$$

This, in terms of Feynman graphs in momentum space, contains many disconnected graphs as well. We choose thus to redefine a connected T product

$$\begin{aligned}\langle T(\psi_n(x)\bar{\psi}_p(x')W_\mu{}^*(y_1)\cdots W_\lambda{}^*(y_m)W_\rho(z_1)\cdots W_\tau(z_{m+1}))\rangle_c \\ =\langle T(\psi_n(x)\bar{\psi}_p(x')W_\mu{}^*(y_1)\cdots W_\lambda{}^*(y_m)W_\rho(z_1)\cdots W_\tau(z_{m+1}))\rangle_0 \\ -\sum_{\text{comb}}\langle T(\psi_n(x)\bar{\psi}_p(x')W_{\nu_i}{}^*(y_i)\cdots W_{\nu_j}{}^*(y_j)W_{\nu_k}(z_k)\cdots)\rangle_c\langle T(W_{\nu_1}{}^*(y_1)\cdots W_{\nu_n}(z_n))\rangle_0,\end{aligned}$$

where $(y_i,\nu_i)(z_k,\nu_k)\cdots$ are any permutation of the original set of indices $(y_1,\mu)(y_2,\nu)\cdots(z_1,\rho)\cdots$, and one is to sum over all combinations. This subtracts out the disconnected graphs. For example, we have

$$\begin{aligned}\langle T(\psi_n(x)\bar{\psi}_p(x')W_\mu{}^*(y_1)W_\tau(z_2))\rangle_c=\langle T(\psi_n(x)\bar{\psi}_p(x')W_\mu{}^*(y_1)W_\rho(z_1)W_\tau(z_2))\rangle_0 \\ -\langle T(\psi_n(x)\bar{\psi}_p(x')W_\rho(z_1))\rangle_0\langle T(W_\mu{}^*(y_1)W_\tau(z_2))\rangle_0-\langle T(\psi_n(x)\bar{\psi}_p(x')W_\tau(z_2))\rangle_0\langle T(W_\mu{}^*(y_1)W_\rho(z_1))\rangle_0.\end{aligned}$$

Baryon number and charge conservation rule out other combinations in the above equation. We have also used the fact that

$$\langle T(\psi_n(x)\bar{\psi}_p(x')W_\rho(z_1))\rangle_c=\langle T(\psi_n(x)\bar{\psi}_p(x')W_\rho(z_1))\rangle_0.$$

With this definition for a connected T product, we have immediately the result that

$$\Box_{y_i}\langle T(\psi_n(x)\psi_p(x')\cdots W_\nu{}^*(y_i)\cdots)\rangle_c=\langle T(\psi_n(x)\bar{\psi}_p(x')\cdots\Box_y W_\nu{}^*(y_i)\cdots)\rangle_c.$$

The surface terms in the full T-product differentiation are absorbed by the disconnected T products.

This means that for connected graphs we need consider only the connected T products of $\psi_n(x)\bar{\psi}_p(x')$ with the current operators. And in particular, we consider the object

$$\frac{\partial}{\partial y_{1_\mu}}\cdots\frac{\partial}{\partial z_{1_\rho}}\cdots\langle T(\psi_n(x)\bar{\psi}_p(x')J_\mu{}^+(y_1)\cdots J_\rho{}^-(z_1)\cdots)\rangle_c.$$

The equation in momentum space follows readily, and is written out in the text.